Period 1, April 28, 2025

35, 36, 45

The function f has first derivative given by $f'(x) = x^4 - 6x^2 - 8x - 3$. On what intervals is the graph of f concave up?

$F''(x) = +$

Ⓐ $(2, \infty)$ only ✔ 2

Ⓑ $(0, \infty)$ 0

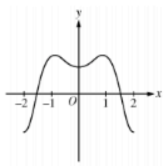
Ⓒ $(-1, 2)$ 0

Ⓓ $(-\infty, -1)$ and $(3, \infty)$ 1

$F'(x) = x^4 - 6x^2 - 8x - 3$

$F''(x) = 4x^3 - 12x - 8$

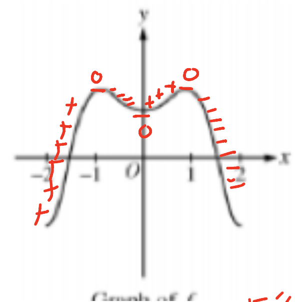
$F''(x) > 0 \quad x > 2$



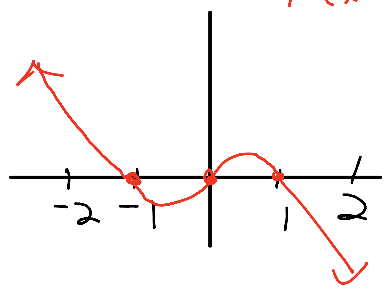
Graph of f

The graph of the function f is shown above for $-2 \le x \le 2$.

Which of the following could be the graph of an ~~antiderivative of f~~?

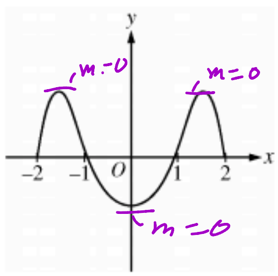


$F'(x) = $ Slope of $F(x)$



The graph of the function f is shown above for $-2 \le x \le 2$.

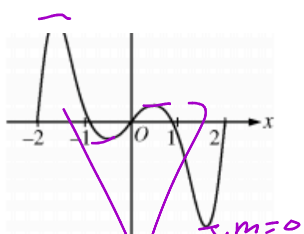
Ⓐ

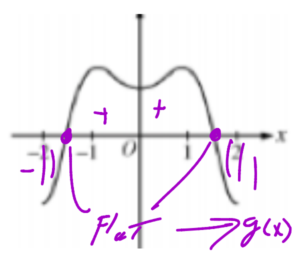


$F'(x) =$

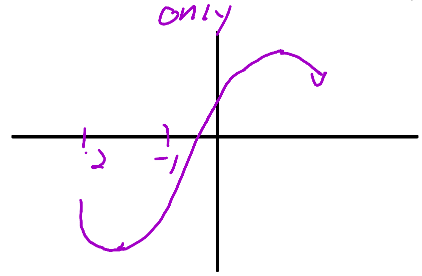
The rate at which water leaks from a tank, in gallons per hour, is modeled by R, a differentiable function of the number of hours after the leak is discovered. Which of the following is the best interpretation of $R'(3)$?

Ⓑ

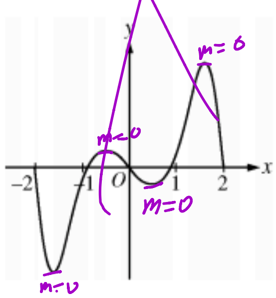




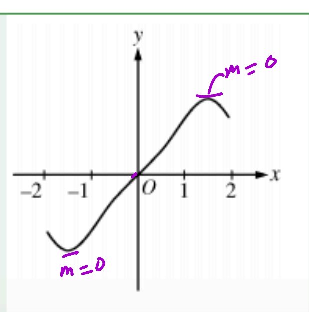
Flat only $\rightarrow g(x) = \int F(x)\,dx$

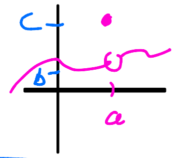


Ⓒ



Ⓓ





$\lim_{x\to a} F(x) = b$

$F(a) = C$

I. If f is defined at $x = a$, then $\cancel{\lim_{x\to a}} f(x) = f(a)$.

II. If f is continuous at $x = a$, then $\boxed{\lim_{x\to a} f(x) = f(a).}$

III. If f is differentiable at $x = a$, then $\lim_{x\to a} f(x) = f(a)$.

also continuous

continious

$\lim_{x\to a} F(x)$ EXISTS

$F(a)$ EXISTS

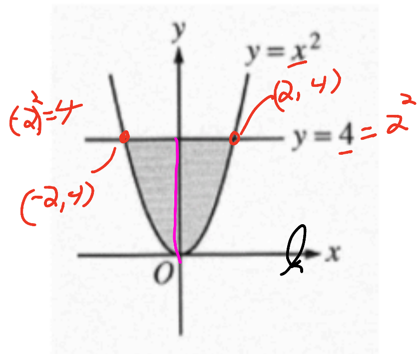
$\lim_{x\to a} F(x) = F(a)$

(A) III only

(B) I and II only

(C) II and III only

(D) I, II, and III



1. The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.

   (a) Find the area of R.

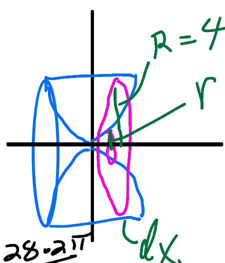
$\int_{-2}^{2} (4-x^2)\,dx = 2\int_{0}^{2} (4-x^2)\,dx = 4x - \frac{1}{3}x^3 \Big|_{-2}^{2}$

$4(2) - \frac{1}{3}(2)^3 - \left[4(-2) - \frac{1}{3}(-2)^3\right]$

$\frac{32}{3} = \frac{48}{3} - \frac{16}{3} = 16 - \frac{16}{3} = 8 - \frac{8}{3} + 8 - \frac{8}{3} = 8 - \frac{8}{3} - \left[-8 + \frac{8}{3}\right]$

   (b) Find the volume of the solid generated by revolving R about the x-axis.

$\frac{256\pi}{5}$



$\pi\int_{-2}^{2} (R^2 - r^2)\,dx = 2\pi\int_{0}^{2} (R^2 - r^2)\,dx$

$2\pi\int_{0}^{2} (4^2 - (x^2)^2)\,dx = 2\pi\left[16x - \frac{1}{5}x^5\right]\Big|_{0}^{2}$

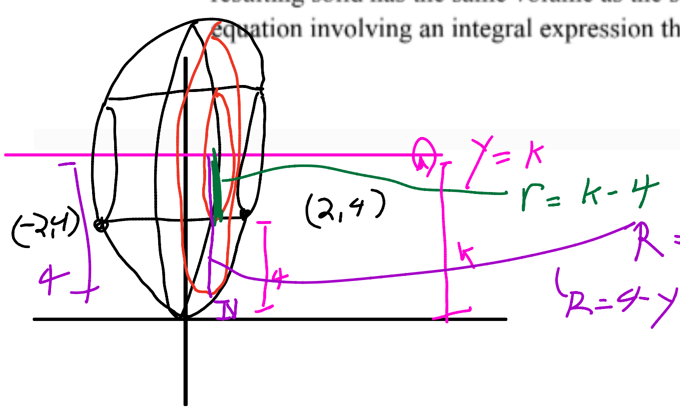
x^4

$2\pi\left[16(2) - \frac{1}{5}(2)^5\right] - \cancel{2\pi\left[16(0) - \frac{1}{5}(0)^5\right]}$

$2\pi\left[32 - \frac{32}{5}\right]$

$2\pi\left[\frac{160}{5} - \frac{32}{5}\right] = \frac{128 \cdot 2\pi}{5}$

   (c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

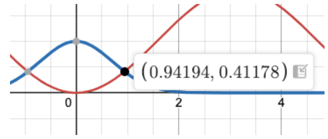


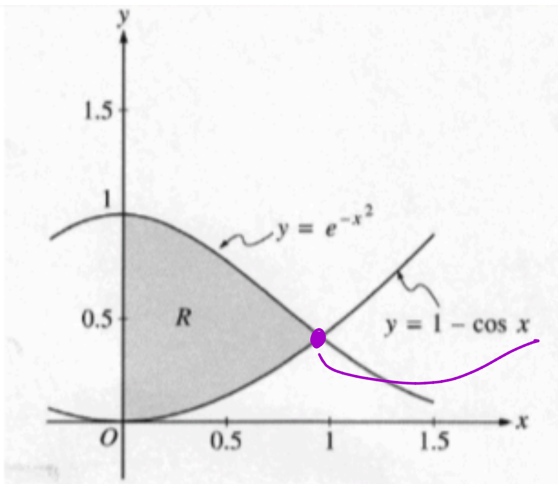
$\pi\int_{-2}^{2} \left[R^2 - r^2\right] dx = \frac{256\pi}{5}$

$\pi\int_{-2}^{2} \left[(k-x^2)^2 - (k-4)^2\right] dx =$

$$\int_0^{0.94194} \left(e^{-x^2} - (1 - \cos x)\right) dx$$

$= 0.590962450123$





$1 - \cos x = e^{-x^2}$

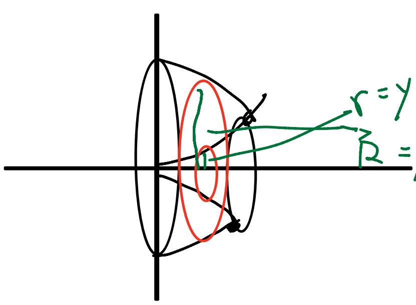
$x = 0.9419$

2. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$ and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

$$\int_0^{0.94194} \left[e^{-x^2} - (1 - \cos x)\right] dx$$

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.



$r = y = 1 - \cos x$

$R = y_1 = e^{-x^2}$

$$\pi \int_0^{0.94194} \left[(e^{-x^2})^2 - (1 - \cos x)^2\right] dx$$

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

$A = (\text{side})^2$

b.

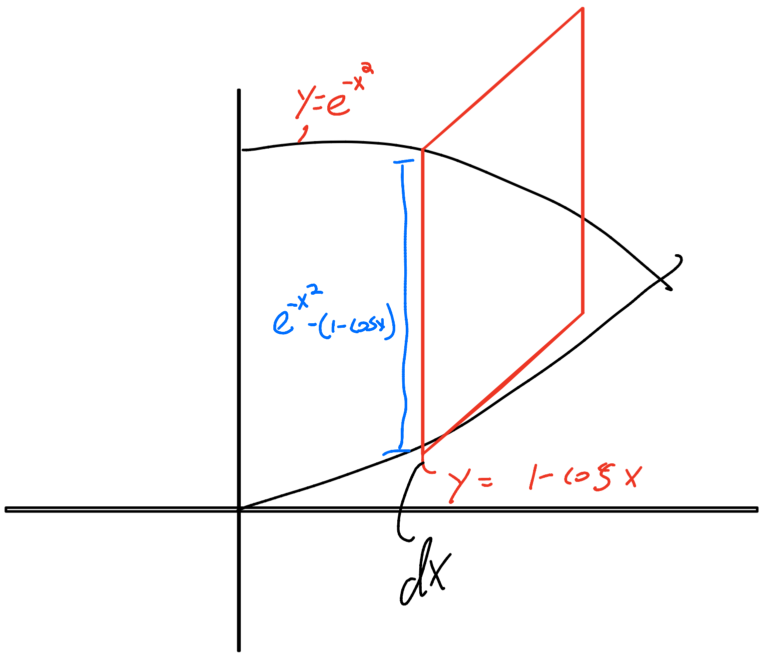
$$\pi \int_0^{0.94194} \left(\left(e^{-x^2}\right)^2 - (1-\cos x)^2\right) dx$$

$= 1.74661409822$

$$\int_0^{0.94194} \left(e^{-x^2} - (1-\cos x)\right)^2 dx$$

$\Downarrow$

$(\text{Side})^2$



continuous

The rate at which water leaks from a tank, in gallons per hour, is modeled by R, a differentiable function of the number of hours after the leak is discovered. Which of the following is the best interpretation of $R'(3)$?

Rate of Rate = Rate of change of Rate

(A) The amount of water, in gallons, that has leaked out of the tank during the first three hours after the leak is discovered

(B) The amount of change, in gallons per hour, in the rate at which water is leaking during the three hours after the leak is discovered

(C) The rate at which water leaks from the tank, in gallons per hour, three hours after the leak is discovered

(D) The rate of change of the rate at which water leaks from the tank, in gallons per hour per hour, three hours after the leak is discovered

A particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \frac{4}{t^3+1}$.

If the position of the particle is $x = 1$ when $t = 2$, what is the position of the particle when $t = 4$?

New position $= 1 + \int_2^4 \frac{4}{T^3+1}\,dT$

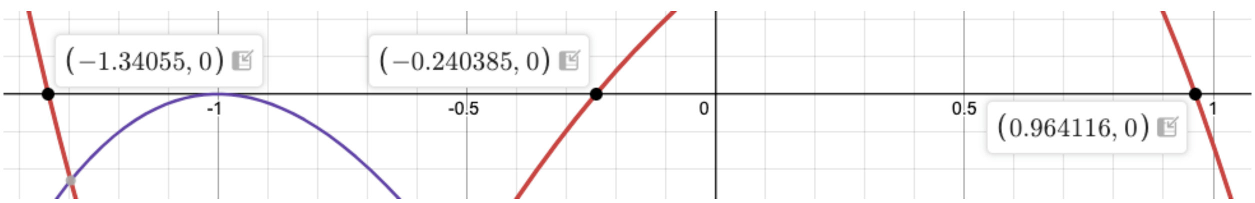
Position $S(T) = \int v(T)\,dT$

$1 + 0.352565$

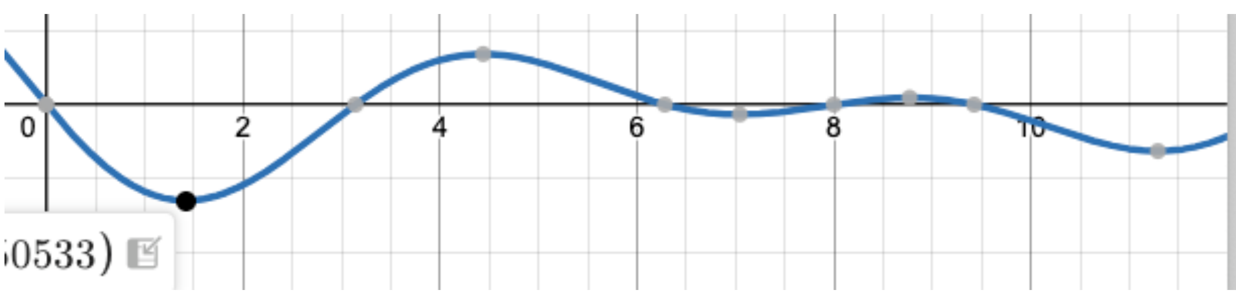
1.352565

$$\int_2^4 \frac{4}{x^3+1}\,dx = 0.352565089963$$

Change From T=2 To T=4

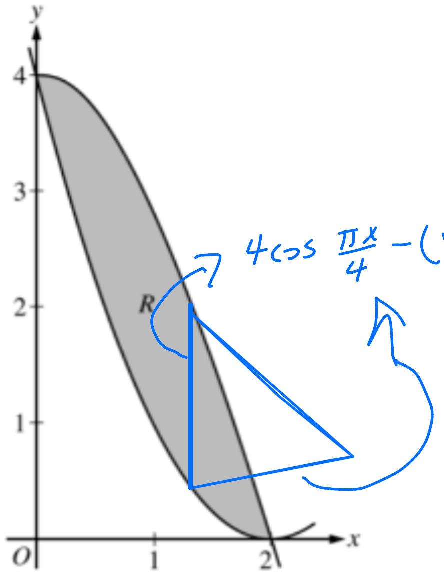


$(-1.5, -1.34055) \cup (-0.240385, 0.964116)$



(1.41998, −6.50533)

$4\cos\frac{\pi x}{4} - (x-2)^2$



$$\frac{1}{2}\int_0^2 \left[4\cos\frac{\pi x}{4} - (x-2)^2\right]^2 dx$$

Let R be the region in the first quadrant bounded by the graphs of $y = 4\cos\left(\frac{\pi x}{4}\right)$ and $y = (x-2)^2$, as shown in the figure above. The region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in region R.

$\frac{1}{2}(\text{side})^2$

$$\frac{1}{2}\int_0^2 \left[4\cos\left(\frac{\pi x}{4}\right) - (x-2)^2\right]^2 dx$$

$= 1.77456261736$

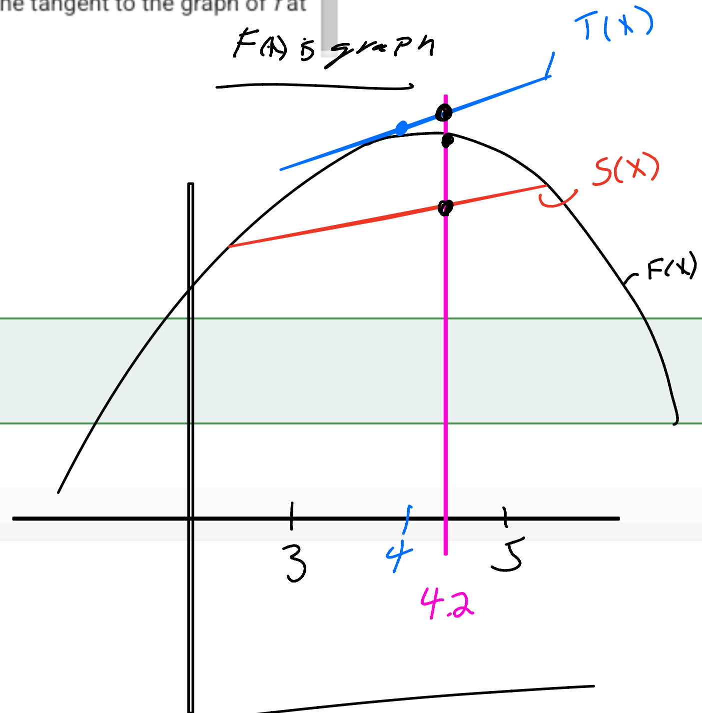
concave Down

Let f be a twice-differentiable function such that $f''(x) < 0$ for all x. The graph of $y = S(x)$ is the secant line passing through the points $(3, f(3))$ and $(5, f(5))$. The graph of $y = T(x)$ is the line tangent to the graph of f at $x = 4$.

Secant Tangent

Which of the following is true?

F(x) is graph



(A) $f(4.2) < S(4.2) < T(4.2)$

(B) $f(4.2) < T(4.2) < S(4.2)$

(C) $S(4.2) < f(4.2) < T(4.2)$

(D) $T(4.2) < f(4.2) < S(4.2)$

x	3	7
$h(x)$	7	22
$h'(x)$	5	10

Selected values of the increasing function h and its derivative h' are shown in the table above.

If g is a differentiable function such that $h(g(x)) = x$ for all x, what is the value of $g'(7)$?

$h(g(3)) = 3$

$h(g(7)) = 7$

$g(7) = h^{-1}(7)$

$g(7) = y = 3$

$h(y) = 7$

$h(3) = 7$

$h(g(x)) = x$

$h^{-1}(x) = g(x)$

$g^{-1}(x) = h(x)$

$g^{-1}(7) = 7 = h(3)$

(A) $-1/10$ 2

(B) $1/10$ 1

(C) $1/5$ ✓ 14

(D) $7/5$ 4

$\cancel{h^{-1}}(\cancel{h}(g(x))) = h^{-1}(x)$

$g(x) = h^{-1}(x)$

$g'(x) = (h^{-1})'(x) = \dfrac{1}{h'(h^{-1}(x))}$

$g'(x) = \dfrac{1}{h'(g(x))}$

$g'(7) = \frac{1}{h'(g(7))}$

$\frac{1}{h'(3)} = \frac{1}{5}$

What is the value of x at which the minimum value of $y = 3x^{\frac{4}{3}} - 2x$ occurs on the closed interval $[0, 1]$?

$\frac{dy}{dx} = 0$ or $\emptyset$

$\frac{4}{3} - 1 = \frac{4}{3} - \frac{3}{3}$

$\frac{dy}{dx} = \cancel{3} \cdot \frac{4}{\cancel{3}} \cdot x^{\frac{1}{3}} - 2$

$\frac{dy}{dx} = 4x^{\frac{1}{3}} - 2 = 0$ or $\emptyset$

$4\sqrt[3]{x} - 2 = 0$

$+2 \quad +2$

$\frac{4\sqrt[3]{x}}{4} = \frac{2}{4}$

$\sqrt[3]{x} = \frac{1}{2}$

$x = \frac{1}{8}$

Min is concave up

$\frac{dy}{dx} = 4x^{\frac{1}{3}} - 2$

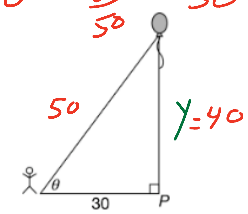
$\frac{d^2y}{dx^2} = 4 \cdot \frac{1}{3} x^{\frac{1}{3} - 1}$

$\frac{4}{3} x^{-\frac{2}{3}} = \frac{4}{3\sqrt[3]{x^2}}$

$\frac{4}{3\sqrt[3]{\left(\frac{1}{8}\right)^2}}$ = Positive

Min

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{30}{50}} = \frac{50}{30}$



A person stands 30 feet from point P and watches a balloon rise vertically from the point, as shown in the figure above. The balloon is rising at a constant rate of 2 feet per second.

What is the rate of change, in radians per second, of angle θ at the instant when the balloon is 40 feet above point P ?

$\frac{dy}{dT} = 2\ FT/S$

Find $\frac{d\theta}{dT}$

$\tan\theta = \frac{y}{30}$

$\sec^2\theta \cdot \frac{d\theta}{dT} = \frac{1}{30} \cdot \frac{dy}{dT}$

$\left(\frac{50}{30}\right)^2 \cdot \frac{d\theta}{dT} = \frac{1}{30} \cdot 2\ FT/sec$

$\left(\frac{5}{3}\right)^2$

$\cancel{\frac{25}{9}} \cdot \frac{d\theta}{dT} = \frac{1}{\cancel{15}\,5} \cdot \frac{\cancel{9}\,3}{25}$

$\frac{d\theta}{dT} = \frac{3}{125}\ Rad/S$

How many vertical asymptotes does the graph of $y = \frac{x-2}{x^4-16}$ have?

$\frac{1}{0}$

One

$\frac{(x-2)}{(x^2-4)(x^2+4)}$

$\frac{(x-\cancel{2})}{(x-\cancel{2})(x+2)(x^2+4)}$ always +

$x+2=0$ always +

$x=-2$

For what value of b does the integral $\int_1^{\cancel{b}\,3} x^2\,dx$ equal $\lim_{n\to\infty}\sum_{k=1}^{n}\underbrace{\left(1+\frac{2k}{n}\right)}^2\frac{2}{n}$?

difference between ~~[illegible]~~

$k=1 \quad n=\infty$

$(1+0)^2 = 1$

$k=n \quad n=\infty$

$\left(1+\frac{2n}{n}\right) = (1+2)^2 = 9$

$(1)^2 = 1$

$(3)^2 = 9$

Question 3

The wind chill is the temperature, in degrees Fahrenheit $(°F)$, a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is $32°F$, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

(a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

(b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

(c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant $32°F$. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

$$W(v) = 55.6 - 22.1 v^{0.16}$$

$$W'(v) = 0 - 22.1(0.16)\, v^{0.16-1} =$$

$$W'(v) = -0.3536\, v^{-.84} = \frac{-0\ 3.536}{v^{0.84}}$$

$$W'(20) = \frac{-3.536}{(20)^{.84}} = -2.85 \text{ or } -2.855$$

Question 3

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.